



III Sem

PG - 820

III  
Second Semester M.Sc. Examination, June 2015

(RNS) (2011-12 and Onwards)

MATHEMATICS

M - 205 : Continuum Mechanics + Fluid Mechanics.

Time : 3 Hours

Max. Marks : 80

**Instructions :** Answer **any five** questions, choosing atleast **one** from each Part. All questions carry **equal** marks.

PART - A

1. a) For an arbitrary vector with components  $b_i$ , if  $a_{ij}b_j$  are components of a vector then show that  $a_{ij}$  are components of a second order tensor. Hence, show that  $\delta_{ij}$  are components of a second-order tensor. 6
- b) Prove that  $\underline{A}$  is a second-order tensor iff it is a linear transformation on vectors and  $a_{ij} = \hat{e}_i \cdot \underline{A} \hat{e}_j$ . 5
- c) If  $\underline{A}$  is an orthogonal tensor such that  $\underline{A} \vec{a} = \vec{a}$  for any vector  $\vec{a}$ , then show that  $\underline{A}^T \vec{a} = \vec{a}$  and the dual vector of skew  $\underline{A}$  is collinear with  $\vec{a}$ . 5
2. a) Define : gradient of a vector, divergence and curl of a tensor. 3
- b) For  $\vec{u} = x_1^2 x_2 \hat{e}_1 + x_2^2 x_3 \hat{e}_2 + x_3^2 x_1 \hat{e}_3$ , verify the identity  $\text{curl } \nabla \vec{u}^T = \nabla \text{curl } \vec{u}$ . 7
- c) State and prove Stokes' theorem for a tensor field. 6

PART - B

3. a) Explain briefly the following :
  - i) Continuum hypothesis.
  - ii) Deformation of arc, surface and volume elements.8

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- b) For the deformation defined by the equations :  
 $x_1 = \alpha x_1^0 + \beta x_2^0$ ,  $x_2 = -\alpha x_1^0 + \beta x_2^0$ ,  $x_3 = \gamma x_3^0$  where  $\alpha$ ,  $\beta$  and  $\gamma$  are positive constants, find  $\underline{F}$ ,  $\underline{F}^{-1}$  and  $J$ . Is the deformation isochoric ? 5
- c) Obtain an expression for Green strain tensor. 3
4. a) Obtain a formula for the material derivative in the spatial form. 3  
 b) Define : path lines and stream lines. Find these lines for the flow defined by the velocity field  $\vec{v} = (1 + at)\hat{e}_1 + x_1\hat{e}_2$  ( $a$  is a constant). Comment on these lines when  $a = 0$ . 7  
 c) Establish Reynolds transport formula and hence deduce that  $\frac{DV}{Dt} = \int_V (\text{tr } \underline{D}) dV$ . 6
5. a) Establish Cauchy's law in the form  $\underline{s}(\hat{n}) = \underline{T}^T \hat{n}$ , where the quantities have their usual meaning. Further, prove that  $\hat{n} \cdot \underline{s}(\hat{n}) = \hat{n}' \cdot \underline{s}(\hat{n}')$  iff  $\underline{T}$  is symmetric. 8  
 b) The stress matrix at a point in a material is given by  $[\tau_{ij}] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ . Find the stress vector at  $P$  acting on the plane element parallel to the plane  $x_1 + 2x_2 + 2x_3 = 0$ . Also, find normal and shear stresses on the element. 8

## PART - C

6. a) Derive the equation of continuity in the Eulerian form from its Lagrangian form. 4  
 b) The stress matrix in a continuum in equilibrium is given by  $[\tau_{ij}] = \begin{bmatrix} x_1^2 & 2x_1x_2 & 0 \\ 2x_1x_2 & x_2^2 & 0 \\ 0 & 0 & x_1^2 + x_2^2 \end{bmatrix}$ . Find the body force acting on the continuum. 5  
 c) Using the appropriate balance law, show that the stress tensor is symmetric. 7



7. a) Establish stress-strain relation for a linear isotropic elastic solid. 6  
b) With usual notations, show that the change in volume of an elastic body in the absence of inertial effects is given by

$$\delta v = \frac{1-2\nu}{E} \left[ \int_V f_i x_i dv + \int_S s_i x_i ds \right]. \quad 5$$

- c) Derive Navier's equation in its standard form. 5
8. a) Prove that every motion of an elastic fluid under conservative body force is circulation preserving. 5  
b) Derive Navier-Stokes equation for a compressible fluid in its usual form. 5  
c) The velocity field  $\vec{v} = K(x_1^2 - x_2^2)\hat{e}_1 - 2K x_1 x_2 \hat{e}_2$  ( $K = \text{constant}$ ) satisfies the Navier-Stokes equation for an incompressible fluid in the absence of body force. Find the pressure distribution. 6

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